Risk Analysis Using Monte Carlo Simulation

Here we present a simple hypothetical budgeting problem for a business start-up to demonstrate the key elements of Monte Carlo simulation. This table shows the setup:

Cost/Budget Item	Input Values	Min. Allowed	10% Likely	Expected Values	90% Likely	Max. Allowed
Administrative	$70,000	$65,000	$67,709	$71,667	$76,097	$80,000
Cost of Sales	$125,000	$120,000	$122,709	$126,667	$131,097	$135,000
Personnel	$335,000	$325,000	$329,950	$336,667	$343,826	$350,000
Professional Fees	$15,000	$12,000	$13,533	$15,667	$17,984	$20,000
Sales & Marketing	$50,000	$45,000	$47,216	$50,000	$52,744	$55,000
Technology	$20,000	$17,000	$18,533	$20,667	$22,984	$25,000

The random-variable probability distribution we have chosen for each of the input budget elements is trangular:

This seems like a reasonable distribution to choose because it's peak represents the initial input values for the budget element variables, that we feel initially are the most likely values; and then it tails off in a linear fashion as we move left toward the "Minimum Allowed Values" and right toward the "Maximum Allowed Values" we initially chose to bracket the range of uncertainty around the initial input values. This triangular distribution results in the "Expected Values," the "Values 10% Likely" and the "Values 90% Likely" shown in the table above.

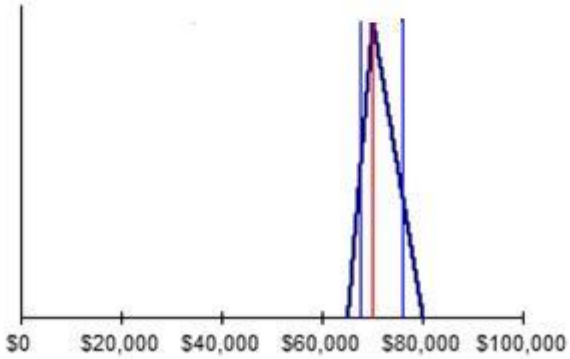
Selecting probability distributions for the input variables in a risk analysis can be tricky. If valid, reliable historical data are available, these should be used as a guide (but it might require doing a bit of research). Otherwise, a combination of common sense and intuition usually works well. Careful inspection of the table above reveals that for most of the input variables, we chose maximum allowed values that are farther above the initial input values than the minimum allowed values are below the initial input values. This is a somewhat conservative approach that helps us to avoid or minimize bad surprises later on due to under-budgeting.

This approach is shown graphically below for each individual budget input variable:

Administrative

Initial Input Value	Min Allowed Value	10% Likely	Expected Value	90% Likely	Max Allowed Value
$70,000	$65,000	$67,709	$71,667	$76,097	$80,000

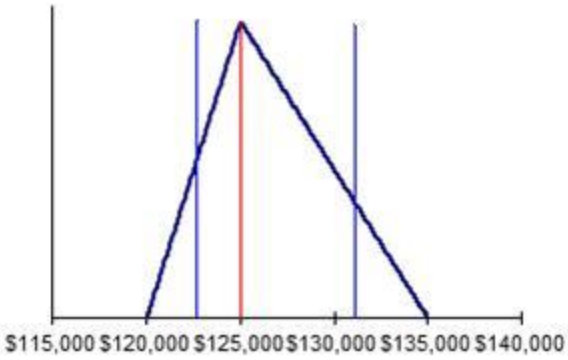
Triangular distribution



Cost of Sales

Initial Input Value	Min Allowed Value	10% Likely	Expected Value	90% Likely	Max Allowed Value
$125,000	$120,000	$122,709	$126,667	$131,097	$135,000

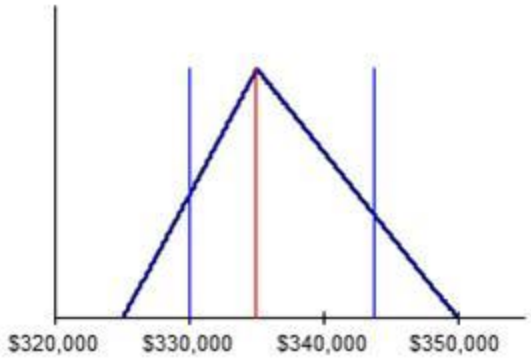
Triangular distribution



Personnel

Initial Input Value	Min Allowed Value	10% Likely	Expected Value	90% Likely	Max Allowed Value
$335,000	$325,000	$329,950	$336,667	$343,826	$350,000

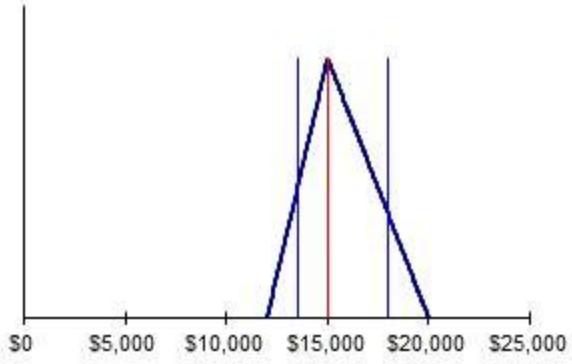
Triangular distribution



Professional Fees

Initial Input Value	Min Allowed Value	10% Likely	Expected Value	90% Likely	Max Allowed Value
$15,000	$12,000	$13,533	$15,667	$17,984	$20,000

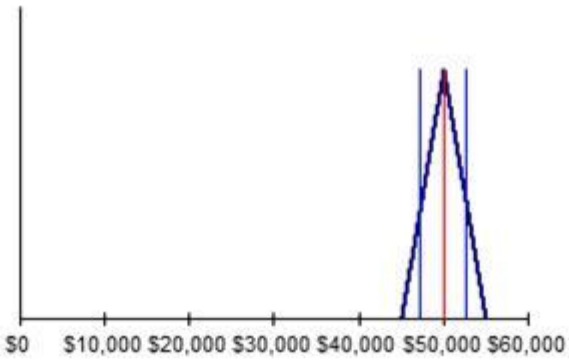
Triangular distribution



Sales and Marketing

Initial Input Value	Min Allowed Value	10% Likely	Expected Value	90% Likely	Max Allowed Value
$50,000	$45,000	$47,216	$50,000	$52,744	$55,000

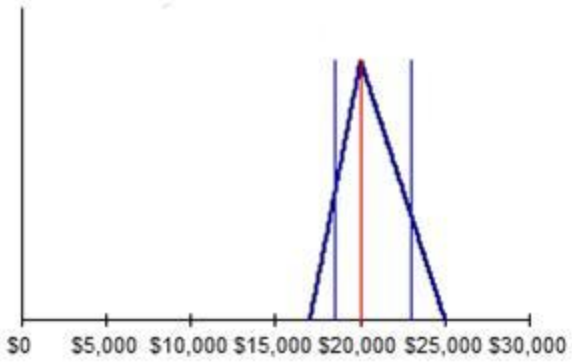
Triangular distribution



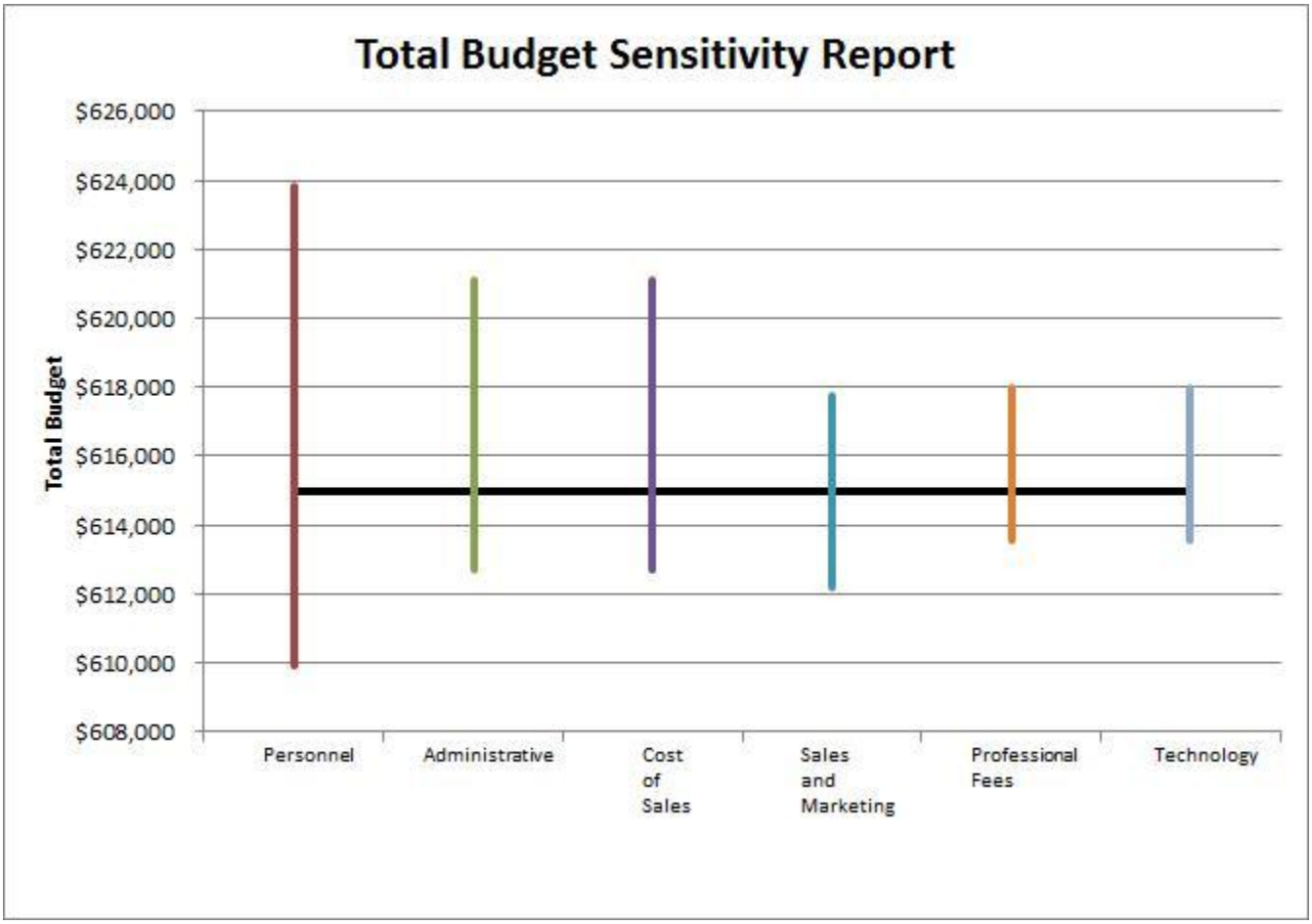
Technology

Initial Input Value	Min Allowed Value	10% Likely	Expected Value	90% Likely	Max Allowed Value
$20,000	$17,000	$18,533	$20,667	$22,984	$25,000

Triangular distribution



Based on the above distributions, we can now generate a Sensitivity Report that shows us the relative influence of each input variable on the Total Budget output variable:



In the graph above, the horizontal line represents the Total Budget number expressed as the sum of the initial budget input variables. A vertical line represents the output range between an input variable's 10% likely and 90% likely values. Inputs with long vertical lines have the most effect on the output variable (Total Budget). Below is a tabular Total Budget sensitivity report:

Total Budget sensitivity to:	10% Likely	Initial	90% Likely	Variance Amount	Variance Cumulative
Personnel	$609,950	$615,000	$623,826	48%	48%
Administrative	$612,709	$615,000	$621,097	17%	65%
Cost of Sales	$612,709	$615,000	$621,097	17%	83%
Sales and Marketing	$612,216	$615,000	$617,744	8%	90%
Professional Fees	$613,533	$615,000	$617,984	5%	95%
Technology	$613,533	$615,000	$617,984	5%	100%

Again we can see that the Personnel Budget has by far the largest influence on the Total Budget

(48%), and the Technology Budget has the smallest influence (5%).

Now we run 1,000 Monte Carlo simulation trials and examine the results:

Minimum result: $600,651	$615,000 Initial Total Budget value
Maximum result: $649,455	20.6% of results are equal or lower
Expected value: $621,314	79.4% of results are equal or greater
Std Deviation: $7,484	

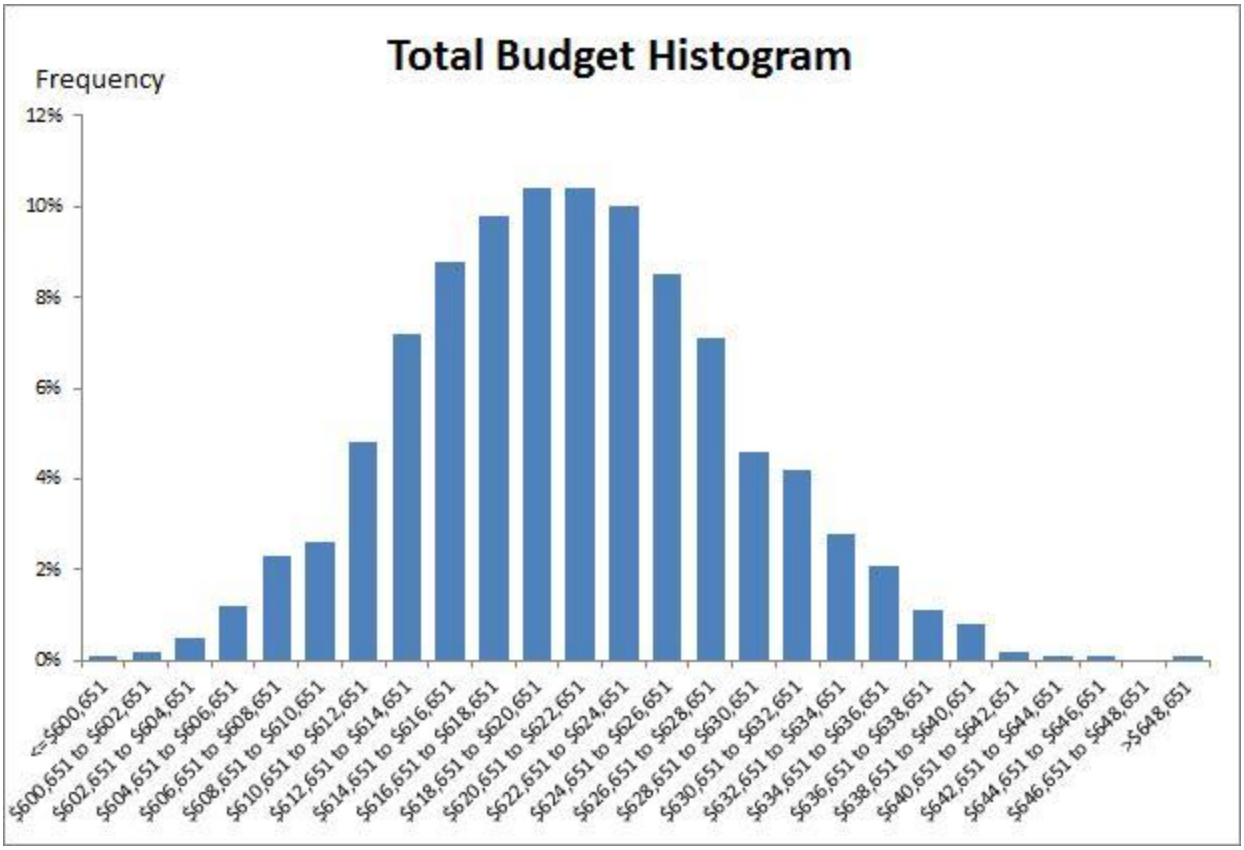
Here is a table showing the percent of simulation values falling below various Total Budget estimates:

5% <= $609,295
10% <= $612,067
15% <= $613,616
20% <= $614,847
25% <= $615,996
30% <= $617,038
35% <= $618,179
40% <= $619,176
45% <= $620,050
50% <= $621,175 (Simulated expected value = $621,314)
55% <= $621,942
60% <= $622,992
65% <= $623,871
70% <= $624,964
75% <= $626,117
80% <= $627,575
85% <= $629,159
90% <= $631,155
95% <= $634,251
100% <= $649,455

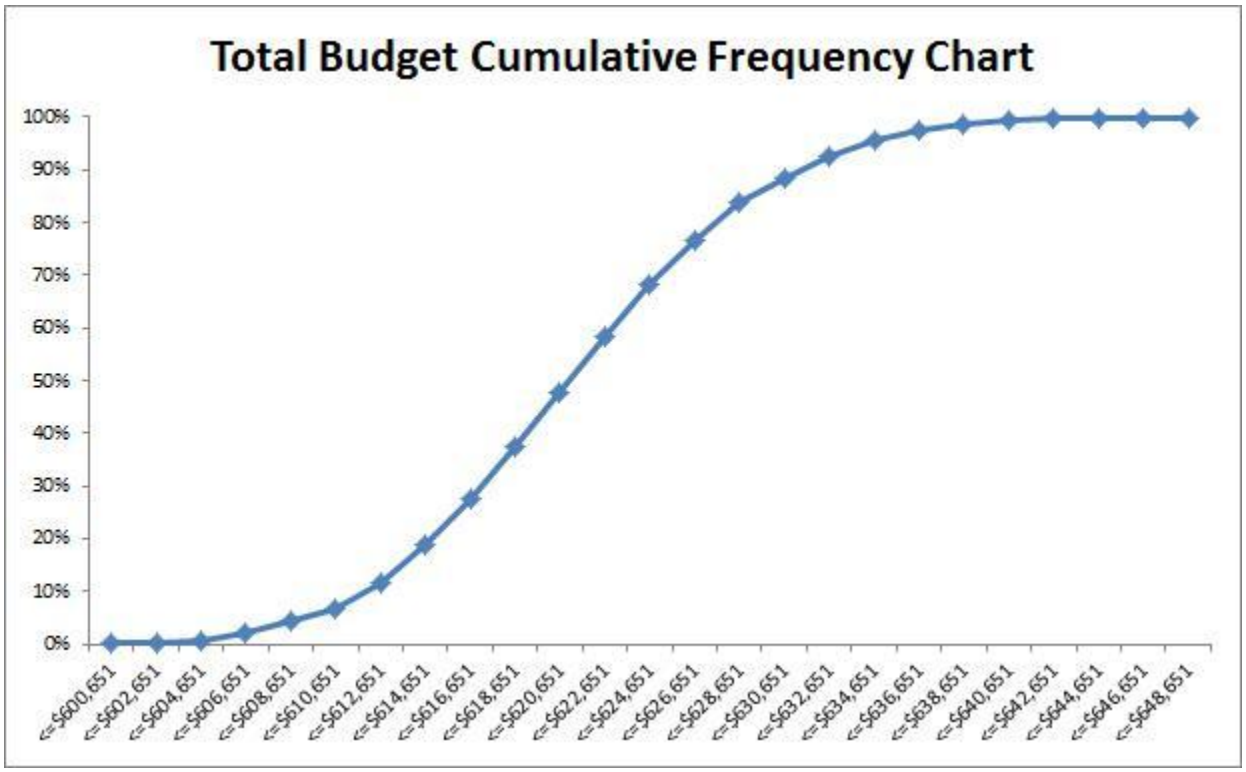
If Total Budget is $615,000:

- 20.6% of results are equal or lower
- 79.4% of results are equal or greater

Here is a histogram of Total Budget simulated values:



Here is the <u>cumulative</u> Total Budget frequency chart:



And here is the tabular version of these distributions:

				Cumulative			Cumulative	
Total Budget	Frequency:			Frequency:			Frequency:	
<=$600,651	1	0.1%	<=$600,651	1	0.1%	=>$600,651	1000	100.0%
$600,651 to $602,651	2	0.2%	<=$602,651	3	0.3%	=>$602,651	997	99.7%
$602,651 to $604,651	5	0.5%	<=$604,651	8	0.8%	=>$604,651	992	99.2%
$604,651 to $606,651	12	1.2%	<=$606,651	20	2.0%	=>$606,651	980	98.0%
$606,651 to $608,651	23	2.3%	<=$608,651	43	4.3%	=>$608,651	957	95.7%
$608,651 to $610,651	26	2.6%	<=$610,651	69	6.9%	=>$610,651	931	93.1%
$610,651 to $612,651	48	4.8%	<=$612,651	117	11.7%	=>$612,651	883	88.3%
$612,651 to $614,651	72	7.2%	<=$614,651	189	18.9%	=>$614,651	811	81.1%
$614,651 to $616,651	88	8.8%	<=$616,651	277	27.7%	=>$616,651	723	72.3%
$616,651 to $618,651	98	9.8%	<=$618,651	375	37.5%	=>$618,651	625	62.5%
$618,651 to $620,651	104	10.4%	<=$620,651	479	47.9%	=>$620,651	521	52.1%
$620,651 to $622,651	104	10.4%	<=$622,651	583	58.3%	=>$622,651	417	41.7%
$622,651 to $624,651	100	10.0%	<=$624,651	683	68.3%	=>$624,651	317	31.7%
$624,651 to $626,651	85	8.5%	<=$626,651	768	76.8%	=>$626,651	232	23.2%
$626,651 to $628,651	71	7.1%	<=$628,651	839	83.9%	=>$628,651	161	16.1%
$628,651 to $630,651	46	4.6%	<=$630,651	885	88.5%	=>$630,651	115	11.5%
$630,651 to $632,651	42	4.2%	<=$632,651	927	92.7%	=>$632,651	73	7.3%
$632,651 to $634,651	28	2.8%	<=$634,651	955	95.5%	=>$634,651	45	4.5%
$634,651 to $636,651	21	2.1%	<=$636,651	976	97.6%	=>$636,651	24	2.4%
$636,651 to $638,651	11	1.1%	<=$638,651	987	98.7%	=>$638,651	13	1.3%
$638,651 to $640,651	8	0.8%	<=$640,651	995	99.5%	=>$640,651	5	0.5%
$640,651 to $642,651	2	0.2%	<=$642,651	997	99.7%	=>$642,651	3	0.3%
$642,651 to $644,651	1	0.1%	<=$644,651	998	99.8%	=>$644,651	2	0.2%
$644,651 to $646,651	1	0.1%	<=$646,651	999	99.9%	=>$646,651	1	0.1%
$646,651 to $648,651	0	0.0%	<=$648,651	999	99.9%	=>$648,651	1	0.1%
>$648,651	1	0.1%						

Here is a summary of the simulations in terms of the amount of change occurring from start to finish for estimated Total Budget values and the standard deviation of the estimates:

# of Simulations	Avg Value	% Change	Std Dev.	% Change
125	$623,170		$7,187	
250	$621,984	-0.2%	$7,512	4.5%
500	$621,766	0.0%	$7,522	0.1%
1000	$621,314	-0.1%	$7,484	-0.5%

We usually stop simulating when the % change drops below 1.0; otherwise we continue. This simulation exercise stopped after 1,000 runs. In our case we could have stopped after 500 runs, but by going longer we were able to reduce the standard deviation slightly.

And finally, here is a calculation of the 95% statistical confidence interval around our simulated expected mean of $321,314:

One-Sample Test

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Expected Total Budget Value	2625.463	999	.000	$621314.4	$620,849.99	$621,778.77

Thus, based on our sample of 1,000 runs, if we were to repeat the simulation many more times using a different random number seed for each simulation, then we would expect 95% of those samples to result in an Expected Total Budget value between $620,849.99 and $621,778.77.

Conclusion

Based on the results of the simulation, we have learned a lot, and we can be more confident in our budget planning. First of all, we learned that we would have only about a 20.6% chance of achieving our initially specified Total Budget of $615,000. If we want to have at least a 50/50 chance of achieving our budget, we should set the budget level at about $621,314. If we want at least a 75% chance of achieving our budget, we should set the budget level at about $626,117. And a budget of about $631,155 would give us an even more comfortable 90% chance of success.