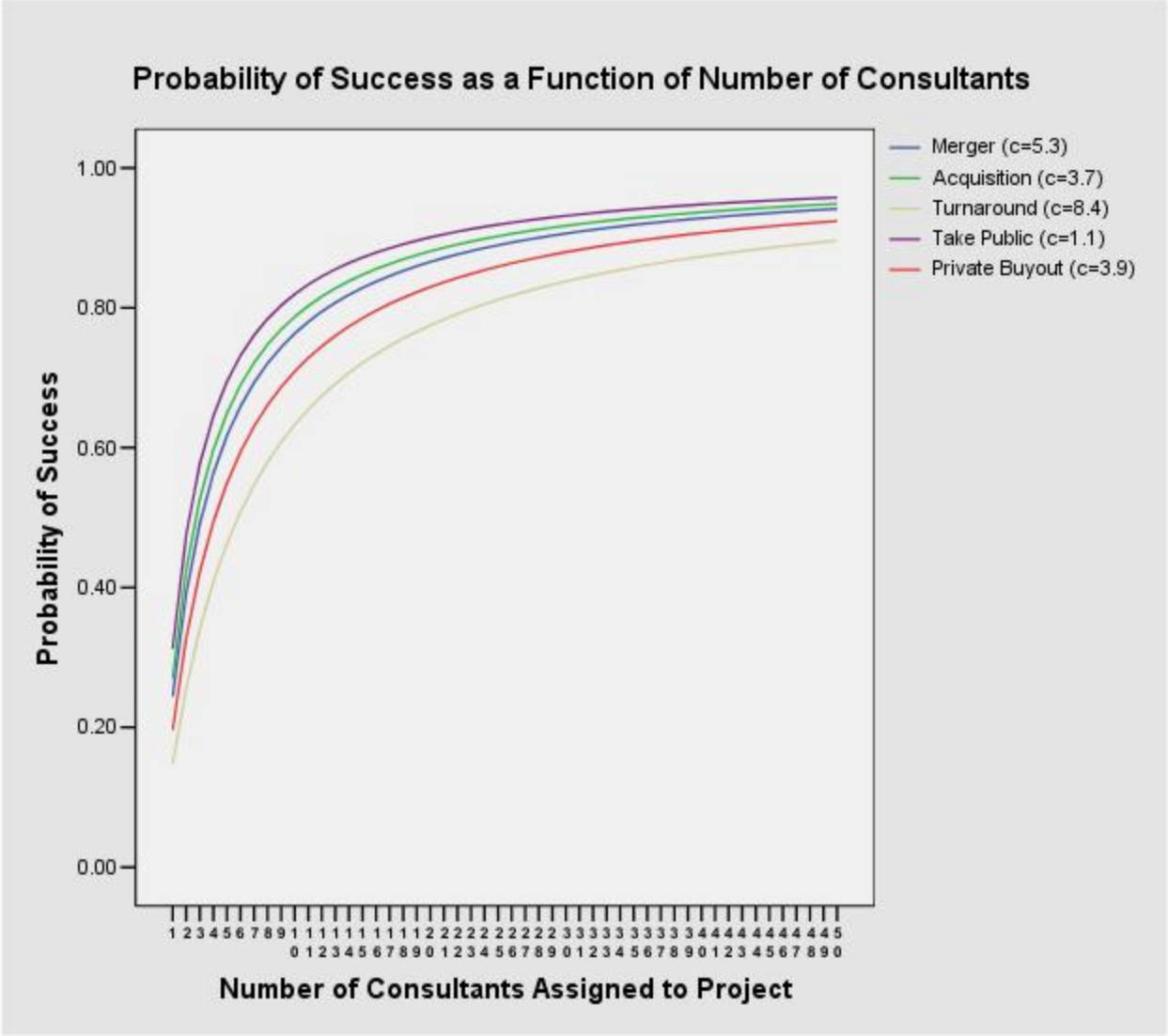
Profit Maximization Via Nonlinear Programming

Here is a hypothetical example that uses nonlinear programming to maximize the expected overall profit on a management consulting firm's project portfolio by optimizing the number of consultants assigned to various projects.

Five management consulting projects have been sold recently:

- A merger
- An acquisition
- A turnaround
- Taking a private company public
- Engineering the buyout and privatization of a public company

A total of 50 consultants are currently available for assignment to the various projects. Each of the five upcoming projects is assigned a probability of success. But this estimated probability is not a simple linear function of the number of consultants assigned to the project. Instead, based on the consulting firm's past experience on similar projects, each type of project is weighted by a constant parameter: X / (X+c), where X is the number of consultants assigned to the project and c is the value of the constant assigned to that project. This graph shows the nonlinear curves that result from this weighting process for the five projects:



Here is the setup of the problem:

Assignment of Consultants to Projects

Consulting Project	Consultants Assigned	Probability Parameter	Success Probability	Profit if Successful	Expected Profit
Merger	10	5.3	0.6536	$750,000.00	$490,196.08
Acquisition	10	3.7	0.7299	$650,000.00	$474,452.55
Turnaround	10	8.4	0.5435	$820,000.00	$445,652.17
Take Public	10	1.1	0.9009	$440,000.00	$396,396.40
Private Buyout	10	3.9	0.7194	$595,000.00	$428,057.55
Total Assigned	50			Total	$2,234,754.76
Total Available	50				

The first column shows the five project types. Initially, as shown in the second column (the model's decision variable), we tentatively assign equal numbers of consultants to each project, with a limit/constraint of 50 consultants in total across all projects. The third column shows the probability constants for each project. The fourth column displays the resulting estimated probability of success. The fifth colum shows the estimated profit for a successful project outcome. We multiply this number by the expected probability of success to arrive at the expected profit, shown in column six.

When we set up the nonlinear programming model, we will stipulate two constraints:

- The total number of consultants assigned across all projects cannot exceed 50 (shown in red)
- Each project must be assigned at least five consultants (not shown in the table)

Also note that based on the starting conditions shown in the table above, the expected total profit across all projects (the objective function, shown in blue) is estimated to be about $2,234,755 if exactly 10 consultants are assigned to each project.

Here is the nonlinear programming optimization result:

Assignment of Consultants to Projects

Consulting Project	Consultants Assigned	Probability Parameter	Success Probability	Profit if Successful	Expected Profit
Merger	12	5.3	0.6936	$750,000.00	$520,231.21
Acquisition	10	3.7	0.7299	$650,000.00	$474,452.55
Turnaround	14	8.4	0.6250	$820,000.00	$512,500.00
Take Public	5	1.1	0.8197	$440,000.00	$360,655.74
Private Buyout	9	3.9	0.6977	$595,000.00	$415,116.28
Total Assigned	50			Total	$2,282,955.79
Total Available	50				

In the table above, column two shows the actual number of consultants assigned to each project. The optimization of consulting project assignments resulted in approximately $48,201 of additional estimated profit over the estimated profit resulting from simply assigning exactly 10 consultants to each project.