
# M/G/S Queueing Model

This hypothetical example shows how a fast-food restaurant can improve its bottom line by installing a self-service soft-drink dispenser. Currently the restaurant's counter staff fill soft-drink orders, which can be time-consuming, especially for large orders. The manager has observed that most patrons don't bother to take advantage of the free refill policy, so she decides to install a self-service soft-drink dispenser at the edge of the seating area, so that customers can fill and refill their own drink orders. She believes that this will save staff time and speed up the waiting line without significantly increasing the cost of goods sold.

Here is the current setup before installing the self-service dispenser:

| Measure | M/M/3 Model | Time Unit |
|---|---|---|
| Customer Arrival Rate | 80 | per hour |
| Customer Service Rate | 30 | per hour |
| Number of Servers | 3 | |
| COV of interarrival times | 1 | |
| COV of service times | 1 | |
| COV of departures | 1.00 | |
| Mean Number in System | 9.05 | |
| Mean Time in System | 6.79 | minutes |
| Mean Number in Queue | 6.38 | |
| Mean Time in Queue | 4.79 | minutes |
| Mean Number in Service | 2.67 | |
| Mean Time in Service | 2.00 | minutes |
| Efficiency | 89% | |

Currently we have a M/M/3 queueing situation. [Note: if you are not familiar with Kendall Notation for queueing models, please read our introductory queueing optimization page before continuing.] There are three queues and three counter staff. The customer arrival rate follows a Poisson distribution, so the interarrival times follow an exponential distribution. The service times also follow an exponential distribution, with most orders being filled fairly quickly, but with some larger orders taking longer. Other staff prepare the food orders that are then picked up by the counter staff. But the counter staff have to fill the drink orders themselves, and the service rate for these also follows an exponential distribution.

In the table above, the "COV of interarrival times" is the coefficient of variation of interarrival times, which is the standard deviation divided by the mean interarrival time. (So, by simple arithmetic, the standard deviation equals the mean, which is a reasonable estimate for such a model.) Similarly, the COV of service times is also 1. The measures highlighted in yellow can change as a function of changes in the gray-highlighted values above them.

By making drink filling and refilling a self-service activity, this variable is removed from the service rate, and the service rate now follows a general probability distribution instead of an exponential one. This is partly due to the fact that the food order is prepared by other staff, and nearly all of it (hamburgers, fries, etc.) is prepared in advance and kept warm; so cashiers need only to grab items and bag them or put them on a tray. This changes the queueing model from a M/M/3 model to a M/G/3 model that is shown below, next to the original model for comparison:

| Measure | M/M/3 Model | M/G/3 Model | Time Unit |
|---|---|---|---|
| Arrival Rate | 80 | 80 | per hour |
| Service Rate per server | 30 | 40 | per hour |
| Number of Servers | 3 | 3 | |
| COV of interarrival times | 1 | 1 | |
| COV of service times | 1 | 0.5 | |
| COV of departures | 1.00 | 0.90 | |
| Mean Number in System | 9.05 | 2.56 | |
| Mean Time in System | 6.79 | 1.92 | minutes |
| Mean Number in Queue | 6.38 | 0.56 | |
| Mean Time in Queue | 4.79 | 0.42 | minutes |
| Mean Number in Service | 2.67 | 2.00 | |
| Mean Time in Service | 2.00 | 1.50 | minutes |
| Efficiency | 89% | 67% | |

As we can see in the table above, the new model now has a smaller standard deviation than the original model, reflecting less variability in service times due to the installation of the self-service drink dispenser. In addition, now the servers are freed up to serve 1/3 more customers in the same amount of time, so the service rate has increased from 30 to 40 customers per hour. So we also see a reduction in average number of customers in the system and the queue, and in time spent waiting in line and average time spent in the entire system.

Theoretically, the less crowded restaurant and shorter waiting lines should attract more customers per hour, once regular customers realize the change that has occurred. So this allows us to create a revised M/G/3 model in which time spent waiting in line increases back to its former (expected maximum) level. But with more efficient service, we see that we can now serve substantially more customers per hour:

| Measure | M/M/3 Model | M/G/3 Model | Time Unit |
|---|---|---|---|
| Arrival Rate | 80 | 113 | per hour |
| Service Rate per server | 30 | 40 | per hour |
| Number of Servers | 3 | 3 | |
| COV of interarrival times | 1 | 1 | |
| COV of service times | 1 | 0.5 | |
| COV of departures | 1.00 | 0.78 | |
| Mean Number in System | 9.05 | 11.85 | |
| Mean Time in System | 6.79 | 6.29 | minutes |
| Mean Number in Queue | 6.38 | 9.02 | |
| Mean Time in Queue | 4.79 | 4.79 | minutes |
| Mean Number in Service | 2.67 | 2.83 | |
| Mean Time in Service | 2.00 | 1.50 | minutes |
| Efficiency | 89% | 94% | |

In reality, traffic may not increase all the way up to 113 customer arrivals per hour, and the time spent waiting in line may not reach its former level. But it's good to know that we can now handle substantially more traffic during peak times; and we may be able to reduce staff slightly, especially at slack times; or at least reduce employee stress by having a less pressured environment. And this change will improve the restaurant's bottom line nicely.