



# Using Linear Programming to Solve a Make-vs.-Buy Decision Problem

Here we present a simple hypothetical example to demonstrate basic Linear Programming optimization concepts. The example involves a company that assembles three types of electric motor. Assembly involves two major steps: winding the motor's armature (winding copper wire repeatedly around a solid iron core), which requires specially trained labor to accomplish properly; and then assembling the completed armature and other parts (field magnet, commutator, brushes, etc.) into a finished motor, which also requires specialized training.

The company has just received an order that is too large to fill using only the company's own labor resources. Therefore it must subcontract one of its competitors to fill the remaining portion of the order.

The following table shows the basic setup of the problem. **Our objective is to fulfill the order at the lowest possible total cost.** The cells containing zeroes are the initially unknown quantities that will be adjusted using Linear Programming. Decision variables are shown in green, constrained variables in red, and the objective in blue:

<b>Electric Motor Assembly</b>				
---- Electric Motor ----				
Number to	Model 1	Model 2	Model 3	
Make	0	0	0	
Buy	0	0	0	
Cost to				
Make	\$52	\$84	\$112	<b>Total Cost</b>
Buy	\$61	\$95	\$125	\$0
# Available	0	0	0	
# Needed	2,800	1,900	1,000	
Hours Required				<b>Used</b>
Armature Wiring	1.5	1	2.5	0
Parts Assembly	1	1.5	1	0
				<b>Available</b>
				9,500
				4,900

Several things can be determined from the project setup table above:

- Motor model 1 is the least expensive to assemble; model 3 is the most expensive
- Outsourcing the labor to the subcontractor is more costly than using in-house labor, and the labor cost differential varies slightly from one motor model to the next
- A total of 5,700 finished motors are needed: 2,800 of model 1; 1,900 of model 2; and 1,000 of model 3
- Because of design differences, each of the motor models requires somewhat different numbers of hours for the armature winding task and the final parts assembly task
- There are a total of 9,500 labor hours available for armature wiring and 4,900 labor hours available for parts assembly in the company that was awarded the contract. (The cost to buy motors from the subcontractor is a fixed price per motor, and the subcontractor has said they can meet whatever demand the contracting company can't fulfill; so we don't need to worry about the subcontractor's labor hours.) Because each of the two assembly tasks is quite different and requires specialized training, specialists are assigned to each task, and one type of specialist can perform only the task for which they were trained

And here is the optimal solution determined by Linear Programming:

<b>Electric Motor Assembly</b>					
---- Electric Motor ----					
Number to	Model 1	Model 2	Model 3		
Make	2,800	733	1,000		
Buy	0	1,167	0		
Cost to					
Make	\$52	\$84	\$112	<b>Total Cost</b>	
Buy	\$61	\$95	\$125	<b>\$430,033</b>	
# Available	2,800	1,900	1,000		
# Needed	2,800	1,900	1,000		
Hours Required				<b>Used</b>	<b>Available</b>
Armature Wiring	1.5	1	2.5	7,433	9,500
Parts Assembly	1	1.5	1	4,900	4,900

The above table tells us several things, not all of which are necessarily intuitively obvious beforehand:

- The contracting company will assemble all of the model 1 and model 3 motors in house; and it will subcontract the assembly of 1,167 model 2 motors to its competitor, while assembling 733 of the model 2 motors in house, at a total project cost of \$430,033
- This will completely fulfill the order, using 7,433 of the contracting company's 9,500 available armature wiring labor hours and all 4,900 available parts assembly labor hours
- Perhaps somewhat counter-intuitively, although the make-vs.-buy cost differential per completed motor is only \$9 for motor model 1, but is \$11 for motor model 2, the company nevertheless can minimize total cost by assembling all model 1 motors in house and outsourcing the assembly of 1,167 model 2 motors

### Solution Report

Here is a brief report summarizing the results of the optimization:

**Objective Cell (Min)**

Name	Original Value	Final Value
Buy Total Cost	\$0	\$430,033

**Variable Cells**

Name	Original Value	Final Value
Make Model 1	0	2,800
Make Model 2	0	733
Make Model 3	0	1,000
Buy Model 1	0	0
Buy Model 2	0	1,167
Buy Model 3	0	0

**Constraints**

Name	Cell Value	Status	Slack
# Available Model 1	2,800	Binding	0
# Available Model 2	1,900	Binding	0
# Available Model 3	1,000	Binding	0
Armature Wiring Used	7,433	Not Binding	2067
Parts Assembly Used	4,900	Binding	0
Make Model 1	2,800	Not Binding	2,800
Make Model 2	733	Not Binding	733
Make Model 3	1,000	Not Binding	1,000
Buy Model 1	0	Binding	0
Buy Model 2	1,167	Not Binding	1,167
Buy Model 3	0	Binding	0

The first two sections of the table above confirm the results we saw previously for the final total cost and the number of motors made in-house and the number bought from the subcontractor. The third section presents results for the model's constraints.

In this third section, the "Cell Value" column confirms the numbers we saw previously. The "Status" column indicates whether or not a particular constraint is binding or not. For the optimal solution, binding constraints are satisfied as strict equalities, whereas nonbinding constraints are not strict equalities. For example, the values for the three electric motor models are all binding, because the customer's order has been filled exactly: the number of motors available as a result of the assembly process is equal to the number needed.

Similarly, the number of hours of specialized labor involved in the parts assembly is binding, because all 4,900 hours of the available specially trained labor will be used. In contrast, only 7,433 of the available 9,500 hours of specialized armature-winding labor will be used if the modeled solution is implemented.

The "Slack" column shows the difference, if any, between the resource constraints and the capacity utilization. The slack value for Armature Winding is 2,067, which is the difference between the number of labor hours used and the number available. The slack value for Parts

Assembly is zero, because all 4,900 available labor hours have been used.

Moving farther down in the table to the nonnegativity conditions imposed on the model (i.e., we cannot use fewer than zero labor hours, and we cannot assemble fewer than zero motors), the slack values indicate how much the decision variables exceed their lower bounds of zero.

## Sensitivity Analysis

Shown below is a sensitivity report for the electric motor solution. This report allows us to determine how sensitive the optimal solution is to adjustments to various model coefficients.

### Variable Cells

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Make Model 1	2800	0.00	52	2	59
Make Model 2	733	0.00	84	11	3
Make Model 3	1000	0.00	112	6	119
Buy Model 1	0	1.67	61	1.E+30	2
Buy Model 2	1167	0.00	95	3	11
Buy Model 3	0	5.67	125	1.E+30	6

### Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
# Available Model 1	2800	59.33	2800	1100	1750
# Available Model 2	1900	95.00	1900	1.E+30	1167
# Available Model 3	1000	119.33	1000	1100	1000
Armature Wiring Used	7433	0.00	9500	1.E+30	2067
Parts Assembly Used	4900	-7.333	4900	1750	1100

For example, we can check to see how much each of the various coefficients could change without changing the optimal solution. Let's examine some of the table columns in the top half of the table (the "Variable Cells") to see how that would work. The first column shows the decision variables; the second column shows the final number of motors assembled to fill the demand; the "Objective Coefficient" column lists the decision variable coefficients as specified in the initial optimization problem setup.

The "Allowable Increase" and "Allowable Decrease" column entries show us how much the coefficients could be increased or decreased without changing the optimal solution, assuming no change in any other coefficients. For example, we could increase the in-house assembly cost of motor model 1 by \$2 or decrease it by \$59 without changing the optimal solution. And we could increase the subcontractor cost of assembling motor model 2 by \$3 or decrease it by \$11 without changing the optimal solution.

Next we'll examine the bottom half of the table ("Constraints"). The "Final Value" column shows us how much of each available resource was used in the motor assembly process. Because we

assembled all needed motors, the first three entries confirm that the final value matches the "Constraint Right Hand (R.H.) Side" value.

And the "Allowable Increase" column value for the "Parts Assembly Used" row tells us that if we had 1,750 hours of additional in-house labor available, we could make all of the motors in-house instead of having to subcontract any of the motor assembly. If we could do this, then we would use 6,650 in-house Parts Assembly labor hours and 8,600 in-house Armature Winding labor hours. We could fulfill the customer's order at a total cost of \$417,200. This represents a cost savings of about \$12,833.

The "Shadow Price" column entries tell us how much the objective function value changes given a one-unit change in the value of the Right Hand value of the constraint, assuming all other constraints remain unchanged. In this column the -7.333 value for "Parts Assembly Used" tells us that for each hour of in-house Parts Assembly labor we add to the project, we would reduce costs by \$7.333. So if we multiply  $\$7.333 \times 1,750$  we arrive at the \$12,833 total cost savings noted above.

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