



## G/G/S Queueing Model

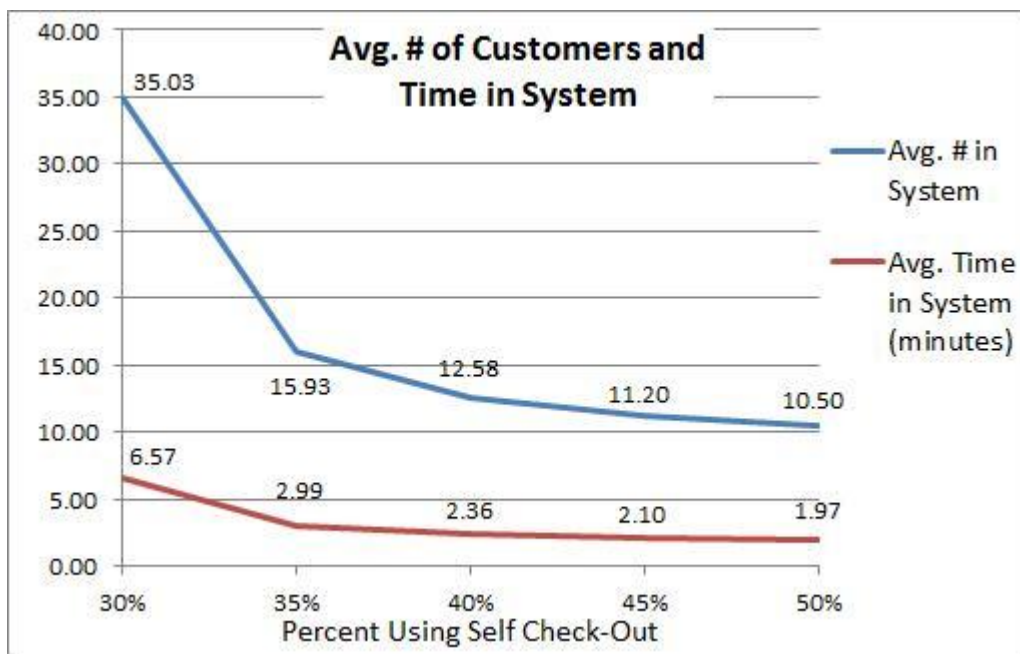
Here is a hypothetical example of a G/G/S queueing model for supermarket check-out lines. [Note: if you are not familiar with Kendall Notation for queueing models, please read our [introductory queueing optimization page](#) before continuing.] This is a G/G/S system because interarrival times and service times both follow a general (nonexponential) random probability distribution.

As the table below indicates, the supermarket has 10 human cashier-served lines and four self check-out stations where the customer scans items from the shopping cart, bags the items and pays at a machine that accepts cash, credit cards or debit cards as payment. Customers enter the store at an average rate of 320 per hour; 70% choose a check-out line served by a human cashier, while 30% choose self-service. Average service time checking out is about 2.6 minutes for cashier-attended lines and about one minute for self check-out lines. (Self check-out lines usually take less time because customers who choose these lines typically have a much smaller number of items as well as less bulky items.)

Measure	Cashiers	Self Check-outs	Overall	Time Unit
Percent of total arrivals	70%	30%		
Arrival rate	224	96	320	per hour
Average interarrival time	0.27	0.63		minutes
Average service time	2.60	1.00		minutes
Arrival rate	3.73	1.60		per minute
Service rate	0.38	1.00		per minute
Servers	10	4		
Utilization	97%	40%		
Traffic intensity	9.71	1.60		
Average # in queue	23.68	0.04	23.73	
Average # in system	33.39	1.64	35.03	
Average time in queue	6.34	0.03	4.45	minutes
Average time in system	8.94	1.03	6.57	minutes
Probability of having to wait	89%	15%	67%	

As currently configured, cashier-served lines average a total check-out time of 8.94 minutes, of which about 6.34 minutes are spent waiting in line behind other customers. The self-serve checkout lines average just over a minute to get through, with almost no waiting in line. Thus, there is an 89% probability of having to wait in line for a human cashier, but only a 15% probability of having to wait for the self check-out stations, for an overall average of 67%.

Management would like to decrease the wait times for human-serviced check-out lines. The self check-out stations have been installed only recently, and many customers probably don't use them because they aren't familiar with them. A queueing model is developed to determine how much impact increased usage of self check-out would have on overall system crowdedness and wait times. As the graph below shows, if the percent of customers using self check-out increased from 30% to 35%, then the overall number of customers in the system and the overall average time in the system would be reduced by more than half:



So a training system is set up whereby store personnel assist customers in learning how to use the stations efficiently. After about a week, 35% of customers are using the self check-out stations. Here is the impact on various measures:

Measure	Cashiers	Self Check-outs	Overall	Time Unit
Percent of total arrivals	65%	35%		
Arrival rate	208	112	320	per hour
Average interarrival time	0.29	0.54		minutes
Average service time	2.60	1.00		minutes
Arrival rate	3.47	1.87		per minute
Service rate	0.38	1.00		per minute
Servers	10	4		
Utilization	90%	47%		
Traffic intensity	9.01	1.87		
Average # in queue	4.97	0.09	5.05	
Average # in system	13.98	1.95	15.93	
Average time in queue	1.43	0.05	0.95	minutes
Average time in system	4.03	1.05	2.99	minutes
Probability of having to wait	66%	20%	50%	

Among other things, we can see that average time spent waiting in line for a human cashier has dropped dramatically, from 6.34 minutes to 1.43 minutes. Continued training manages to boost the percent using self check-out to 40%, with the following results:

Measure	Cashiers	Self check-outs	Overall	Time Unit
Percent of total arrivals	60%	40%		
Arrival rate	192	128	320	per hour
Average interarrival time	0.31	0.47		minutes
Average service time	2.60	1.00		minutes
Arrival rate	3.20	2.13		per minute
Service rate	0.38	1.00		per minute
Servers	10	4		
Utilization	83%	53%		
Traffic intensity	8.32	2.13		
Average # in queue	1.96	0.16	2.13	
Average # in system	10.28	2.30	12.58	
Average time in queue	0.61	0.08	0.40	minutes
Average time in system	3.21	1.08	2.36	minutes
Probability of having to wait	48%	25%	39%	

Once again we have managed to cut waiting times in the cashier-serviced lines by more than half; and there is little negative impact on the self check-out lines. The store manager believes, based on the line graph above, that there is little to be gained from trying to increase the percent of customers using self check-out beyond about 40%; and the effort might not be very successful anyway, because there are a limited number of customers having grocery carts with few items and/or few bulky items.

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