



## Decision Analysis

Formal **decision analysis** is often used when a decision must be made under conditions of significant uncertainty. SmartDrill can assist management with any of a variety of decision analysis approaches, depending on the situation.

Here we discuss various possible approaches to decision analysis, as well as their strengths and limitations, using a hypothetical example of a company that is trying to decide where to purchase land on which to build a new restaurant. The decision is based on where the company believes a new sports stadium will be built, because the presence of the stadium would greatly increase the restaurant's profitability.

Based on existing research, the company knows that the city will build the new sports stadium in either the eastern or western suburb of the city, but the city has not made a final decision. The company needs to buy the land before the stadium siting decision is made public, in order to save money on the purchase price of the land, which will probably increase substantially when the location for the new stadium is made public.

Here is the statement of the problem:

### Restaurant Land Purchase Decision

|   | Parcel of land located in: |      |
|---|----------------------------|------|
|   | East                       | West |
| Current price   | \$30                       | \$21 |
| Value if stadium is built at this location            | \$52                       | \$40 |
| Value if stadium is <i>not</i> built at this location | \$10                       | \$7  |

(Values are in millions of dollars)

A well-situated parcel of land in the eastern suburb currently costs \$30 million; a parcel in the western suburb currently costs \$21 million. The present value of future cash flows of the East location is \$52 million if the stadium is built there; the value of the West location is \$40 million if

the stadium is built there. The value of the East parcel if the stadium is not built there is only \$10 million; and the corresponding value for the West parcel is \$7 million.

### Using a Decision Matrix

One common method of portraying decision analysis involves decision matrices. Here is the basic **payoff matrix** for this problem:

| Restaurant Land Purchase Decision |                              |        |
|-----------------------------------|------------------------------|--------|
| Land Purchased<br>at Location(s)  | Stadium is Built at Location |        |
|                                   | East                         | West   |
| East                              | \$22                         | (\$20) |
| West                              | (\$14)                       | \$19   |
| East and West                     | \$8                          | (\$1)  |
| Neither                           | \$0                          | \$0    |

The table above shows us that if the company buys the East parcel and the stadium is built there, the expected payoff is \$22 million. If they buy the West parcel and the stadium is built in the East, then they will lose \$20 million. If they buy the West parcel and the stadium is built there, then the payoff will be \$19 million. And if they buy the West parcel and the stadium is built in the East, then they will lose \$14 million.

The company could also hedge its bets and buy both parcels of land. If they do this and the stadium is built in the East, then the payoff is \$8 million; but if the stadium is built in the West, then they lose \$1 million. And of course, if the company decides not to buy either parcel, then they will neither gain nor lose anything.

So the question is: What should the company do? To answer this question, there are several **decision rules** that they could consider. Let's examine these various rules.

## Maximax

If the company is optimistic and believes that events will tend to favor their decision, then they could use the **maximax decision rule**, as shown below:

| Restaurant Land Purchase Decision |                              |        |      |
|-----------------------------------|------------------------------|--------|------|
| Land Purchased at Location(s)     | Stadium is Built at Location |        |      |
|                                   | East                         | West   | MAX  |
| East                              | \$22                         | (\$20) | \$22 |
| West                              | (\$14)                       | \$19   | \$19 |
| East and West                     | \$8                          | (\$1)  | \$8  |
| Neither                           | \$0                          | \$0    | \$0  |

Here the assumption is that the company should make the decision that has the **maximum payoff**. Thus they would buy the East parcel and hope to make \$22 million. But of course there is no guarantee that the company actually will have made the right choice, and they could lose \$20 million if they're wrong. And what if the \$20 million loss in the above table were actually a much larger loss, e.g., \$100 million? Unfortunately, the maximax rule doesn't take into account the magnitude of the downside of a wrong decision; so it can lead to some really poor decisions.

## Maximin

If the company is conservative and pessimistic, and believes that events will tend to go against their decision, then they could use the **maximin decision rule**, as shown below:

| Restaurant Land Purchase Decision |                              |        |        |
|-----------------------------------|------------------------------|--------|--------|
| Land Purchased at Location(s)     | Stadium is Built at Location |        |        |
|                                   | East                         | West   | MIN    |
| East                              | \$22                         | (\$20) | (\$20) |
| West                              | (\$14)                       | \$19   | (\$14) |
| East and West                     | \$8                          | (\$1)  | (\$1)  |
| Neither                           | \$0                          | \$0    | \$0    |

Here the company is aiming for the least loss. In this case, that would mean that the company wouldn't buy either parcel of land. This decision rule can also lead to some poor decisions, because it doesn't consider the magnitude of the potential upside. For example, what if the \$22 million cell in the above table were actually \$100 million?

## Minimax Regret

Another approach to the problem involves using the **minimax regret decision rule**, that focuses on what is sometimes referred to as **opportunity cost** or **opportunity loss**. Here the strategy is to make the decision that, if wrong, will result in the least amount of regret or opportunity loss, as shown below:

| Restaurant Land Purchase Decision |                              |      |      |
|-----------------------------------|------------------------------|------|------|
| Land Purchased<br>at Location(s)  | Stadium is Built at Location |      |      |
|                                   | East                         | West | Max  |
| East                              | \$0                          | \$39 | \$39 |
| West                              | \$36                         | \$0  | \$36 |
| East and West                     | \$14                         | \$20 | \$20 |
| Neither                           | \$22                         | \$19 | \$22 |

A green arrow points from the word "minimum" to the \$20 value in the "East and West" row, "Max" column.

So if the company buys the East parcel, but the stadium is built in the West, then the maximum regret or opportunity loss would be \$39 million. If they buy the West parcel, but the stadium is built in the East, then the maximum opportunity loss would be \$36 million. If they buy both East and West parcels, then the maximum regret would be \$20 million. And if they decide to purchase neither parcel, then the opportunity loss would be \$22 million. So if they pick the minimum of these four maximum regret values (\$20 million), they would decide to buy both East and West parcels.

This decision rule is arguably the most defensible of the various decision rules we have considered so far. But it can also lead to some poor decisions if just one more option is added to the problem. For example, suppose a third attractive parcel of land suddenly becomes available in either the East or West suburb. Depending on the values assigned to this third parcel, it is possible, according to the conditions of the minimax regret rule, that even if the newly available third parcel is not chosen, **simply applying the rule to this new situation could actually change the original decision**. So changes in the situation can lead to some inconsistent results, which can be quite troubling and problematic for the decision-maker.

## **Expected Value**

The previous examples of decision matrices were **nonprobabilistic**, because they assumed that the stadium was equally likely to be built in either the East or the West suburb. But in many cases where historical data exist, or research can be conducted, or the services of an experienced expert consultant can be contracted, we can try to assign probabilities to the possible stadium siting outcomes, thus creating a **probabilistic analysis**. In our example above, we implicitly assigned probabilities of 0.50 to each possible reality (the stadium's being built in either the East or West). But we can use explicit probabilities in what is called **expected value** or **expected monetary value (EMV) analysis**.

First, here is the original payoff matrix, to which we have added probabilities for where the stadium is likely to be built:

| Restaurant Land Purchase Decision |                              |        |       |
|-----------------------------------|------------------------------|--------|-------|
| Land Purchased at Location(s)     | Stadium is Built at Location |        | EMV   |
|                                   | East                         | West   |       |
| East                              | \$22                         | (\$20) | \$1.0 |
| West                              | (\$14)                       | \$19   | \$2.5 |
| East and West                     | \$8                          | (\$1)  | \$3.5 |
| Neither                           | \$0                          | \$0    | \$0.0 |
| Probability                       | 0.5                          | 0.5    |       |

← maximum

Here we have simply multiplied the original payoff values by the probability of the respective states of nature, and then summed across each row. For example, for the first row we multiply  $22 \times 0.50$  to get 11; and we multiply  $-20 \times 0.50$  to get -10. When we sum these two products, we get the 1.0 shown in the EMV column.

Note that the EMV solution to the decision problem gives us the same result as the previous minimax regret solution: the company chooses to buy both parcels of land to hedge their bets. But we can also transform the EMV approach into an **expected regret** or **expected opportunity loss (EOL)** approach, just as we did previously for the nonprobabilistic minimax regret matrix. Here is the result:

| Restaurant Land Purchase Decision |                              |      |        |
|-----------------------------------|------------------------------|------|--------|
| Land Purchased at Location(s)     | Stadium is Built at Location |      | EOL    |
|                                   | East                         | West |        |
| East                              | \$0                          | \$39 | \$19.5 |
| West                              | \$36                         | \$0  | \$18.0 |
| East and West                     | \$14                         | \$20 | \$17.0 |
| Neither                           | \$22                         | \$19 | \$20.5 |
| Probability                       | 0.5                          | 0.5  |        |

← minimum

Here we see that the EMV and EOL methods lead us to the same decision. And, in fact, these two approaches will always arrive at the same decision. They are two equivalent ways of choosing the same decision alternative.

## Sensitivity Analysis

Now let us examine one of the particular benefits of the probabilistic EMV decision analysis. Sometimes, based on historical data, additional research and/or expert advice, we can adjust the probabilities away from 0.50 to reflect this new information. And sometimes relatively small adjustments can make a big difference in our final decision.

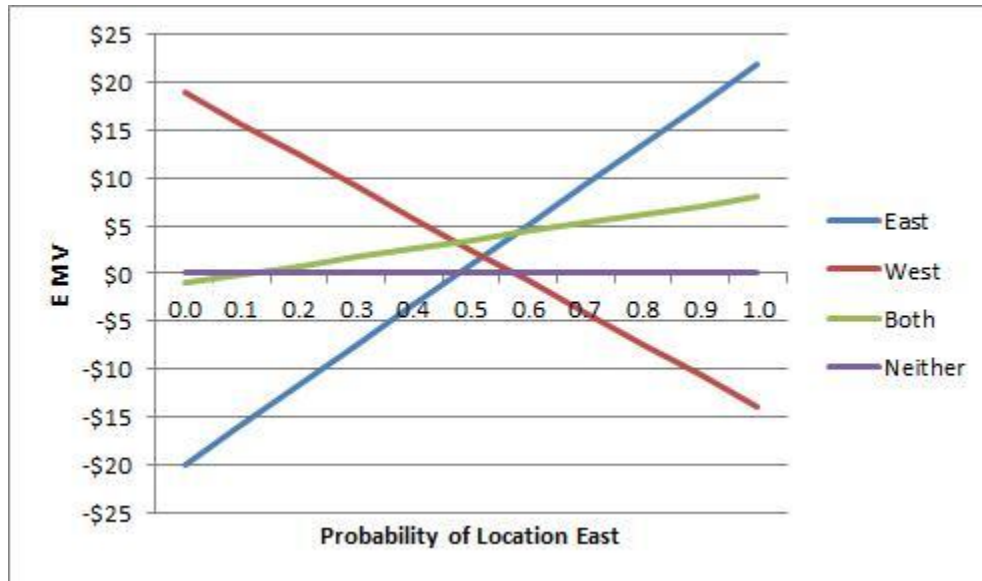
Below is a **sensitivity analysis** table that shows us how the decision would change based on a change in the probabilities of the two possible realities or states of nature:

|                               | EMV East     | EMV West     | EMV Both     | EMV Neither  |
|-------------------------------|--------------|--------------|--------------|--------------|
| <b>Prob. of Location East</b> | <b>\$1.0</b> | <b>\$2.5</b> | <b>\$3.5</b> | <b>\$0.0</b> |
| 0.0                           | -20          | 19           | -1           | 0            |
| 0.1                           | -15.8        | 15.7         | -0.1         | 0            |
| 0.2                           | -11.6        | 12.4         | 0.8          | 0            |
| 0.3                           | -7.4         | 9.1          | 1.7          | 0            |
| 0.4                           | -3.2         | 5.8          | 2.6          | 0            |
| 0.5                           | 1            | 2.5          | 3.5          | 0            |
| 0.6                           | 5.2          | -0.8         | 4.4          | 0            |
| 0.7                           | 9.4          | -4.1         | 5.3          | 0            |
| 0.8                           | 13.6         | -7.4         | 6.2          | 0            |
| 0.9                           | 17.8         | -10.7        | 7.1          | 0            |
| 1.0                           | 22           | -14          | 8            | 0            |

In the table above, the first row of numbers are simply the EMV values we saw before that caused us to buy both parcels of land. The leftmost column of numbers are a range of probabilities for the stadium's being built at the East location. Notice how the EMVs for the various possible decisions change as we change these probabilities. Our original problem assumed a 50/50 chance of the stadium's being built at the East location, and the EMVs for this scenario are shown in the row highlighted in red.

But if the probabilities begin to deviate enough from the original 50/50 scenario, then we see shifts in the decision choice. The rows of the table highlighted in green show the range of probabilities that would lead us to choose to buy only the West parcel; and the rows highlighted in blue show us the range of probabilities that would lead us to choose to buy only the East parcel.

The line graph below gives us another way to visualize the data in the table:



So now let's see what happens if the probability of the stadium's being built at the East location changes from 0.50 to 0.45. Although this is a small change in probability, it ends up changing the decision. Now, instead of buying both parcels of land, the company chooses to buy just the West parcel, thus saving a substantial amount of investment capital:

| <b>Restaurant Land Purchase Decision</b>     |                              |        |         |
|--|------------------------------|--------|---------|
| Land Purchased<br>at Location(s)             | Stadium is Built at Location |        | EMV     |
|  | East                         | West   |         |
| East   | \$22                         | (\$20) | (\$1.1) |
| West   | (\$14)                       | \$19   | \$4.2   |
| East and West                                | \$8                          | (\$1)  | \$3.1   |
| Neither                                      | \$0                          | \$0    | \$0.0   |
| Probability                                  | 0.45                         | 0.55   |         |
| Expected Payoff Using<br>Perfect Information | \$22                         | \$19   | \$20.4  |
|  |                              | EVPI = | \$16.2  |

A green arrow points from the text "maximum" to the EMV value of \$4.2 for the "West" option.

## The Expected Value of Perfect Information

So the additional data, research and/or expert advice has allowed us to adjust the probabilities, which has in turn allowed us to change our decision with confidence. But how much should we be willing to pay for the new data, research and/or expert advice, which is sometimes referred to as the **expected value of perfect information**?

In the table above we have subtracted the maximum EMV from the "EMV with perfect information" to arrive at the value of perfect information, which in this case is about \$16.2 million. [Note: Given the same input probability data, the expected value of perfect information (EVPI) will always be equal to the minimum expected opportunity loss (EOL) that we saw previously.]

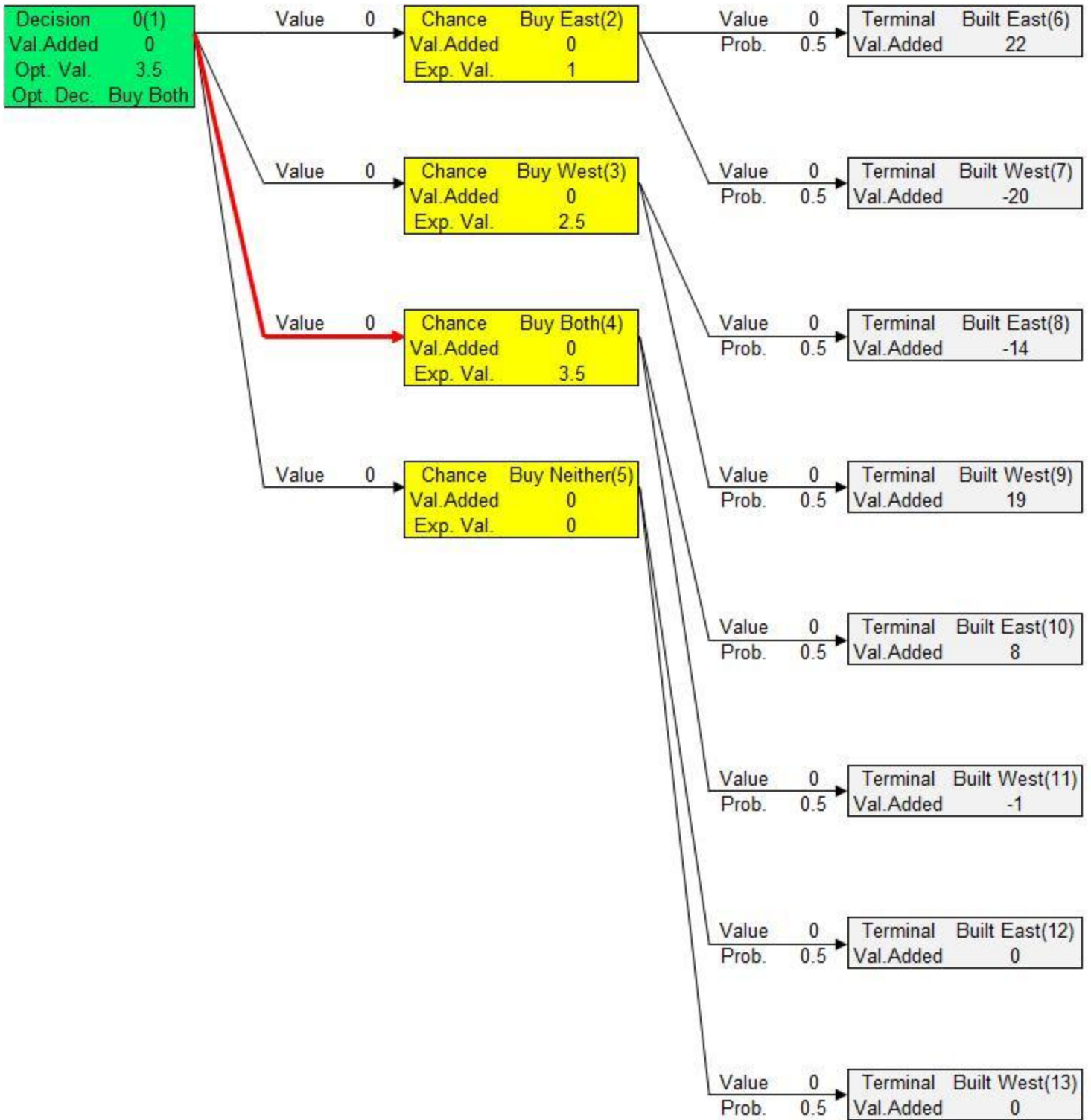
## Decision Trees

Another general method of portraying decision analyses is via the use of decision trees instead of decision matrices. The various nonprobabilistic decision rules that we discussed previously can be applied to decision trees as well as to payoff matrices. However, decision trees are most commonly used to portray the results of either the probabilistic EMV decision rule or rules based on **utility theory** (which is discussed below).

Decision trees have some advantages over decision matrices. First, the graphical nature of decision trees makes it very easy to identify the decision alternative having the highest EMV; and decision trees can clearly portray much more complicated decision situations than can be easily understood from a matrix. In addition, it is easy to "roll back" complicated decision trees and then show analyses based not only on different probabilities but also on different theoretical analytic approaches (such as EMV rules vs. utility theory). And lastly, a big advantage of decision trees is that they can easily portray **multi-stage decision processes**, where a subsequent decision may depend on earlier ones in a chain of sequential decisions. Such situations arise very often, making decision trees a more attractive method of displaying results instead of via matrices.

For the restaurant siting problem, the decision tree below shows us the various combinations of decision alternatives (where to buy land) and possible states of nature (where the stadium could be built), and shows us the outcomes clearly:





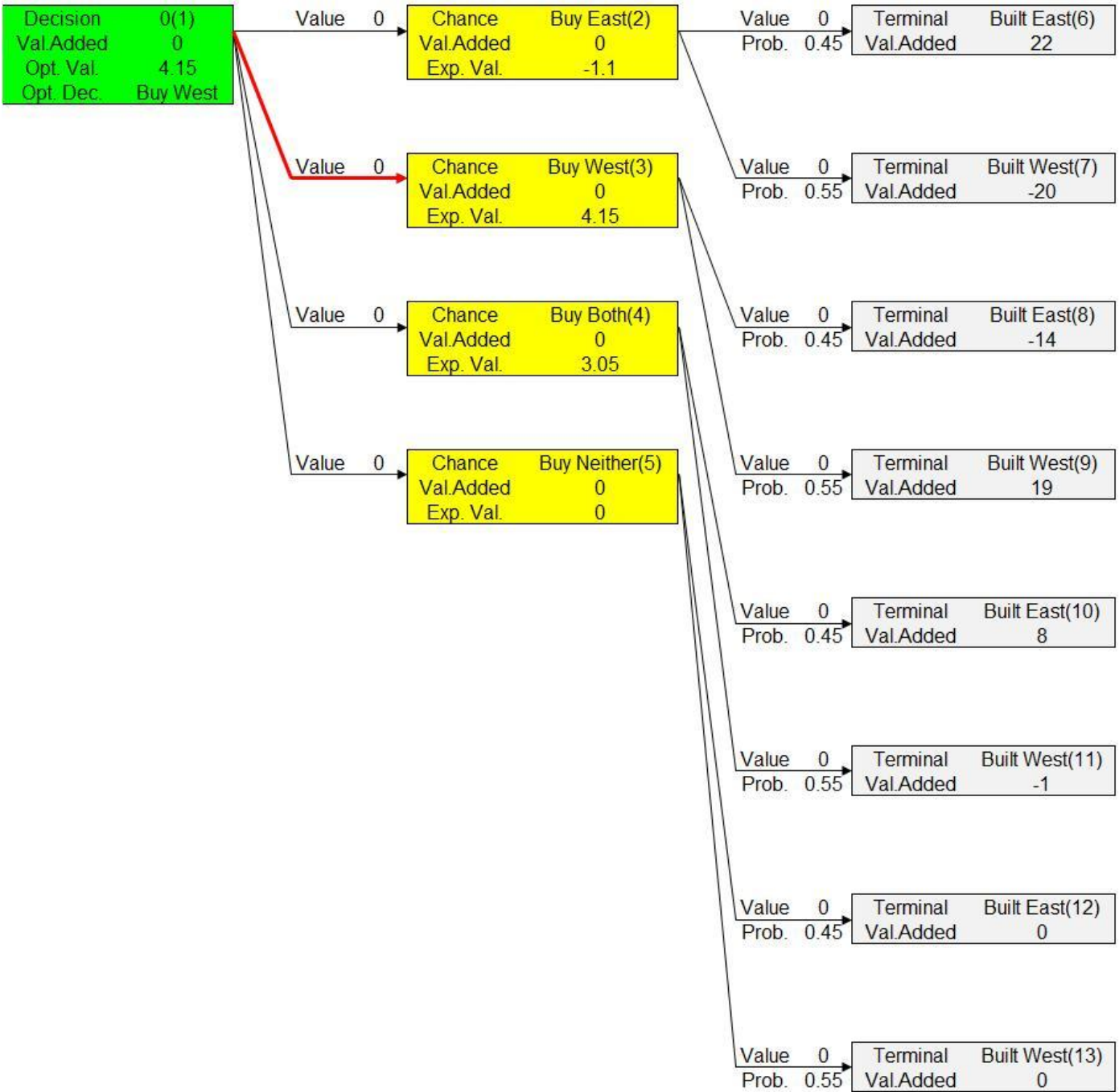
The above decision tree shows us what decision is recommended when the states of nature have equal probabilities of occurring (the green box). The yellow boxes represent the possible decision alternatives; and the gray boxes represent the two possible states of nature that can be

combined with the four decision alternatives to give us eight possible combinations.

The green box is labeled "Decision" because its text tells us the final decision ("Buy Both") as well as the EMV associated with that decision alternative. The yellow boxes are labeled "Chance" because we do not know what the result of any given decision will be until we learn where the stadium is actually going to be built. The gray boxes are labeled "Terminal" because they represent the possible states of reality that end the decision analysis process in the tree diagram.

Notice also that the yellow boxes contain the EMV for each choice; and the 0.50 probability is shown to the left of each of the Terminal nodes, reflecting the fact that each state of reality is assumed to have an equal probability. The Terminal boxes also contain the same payoff numbers we saw in the original payoff matrix near the top of this Web page. And the red arrow points clearly to the decision alternative that the model has chosen.

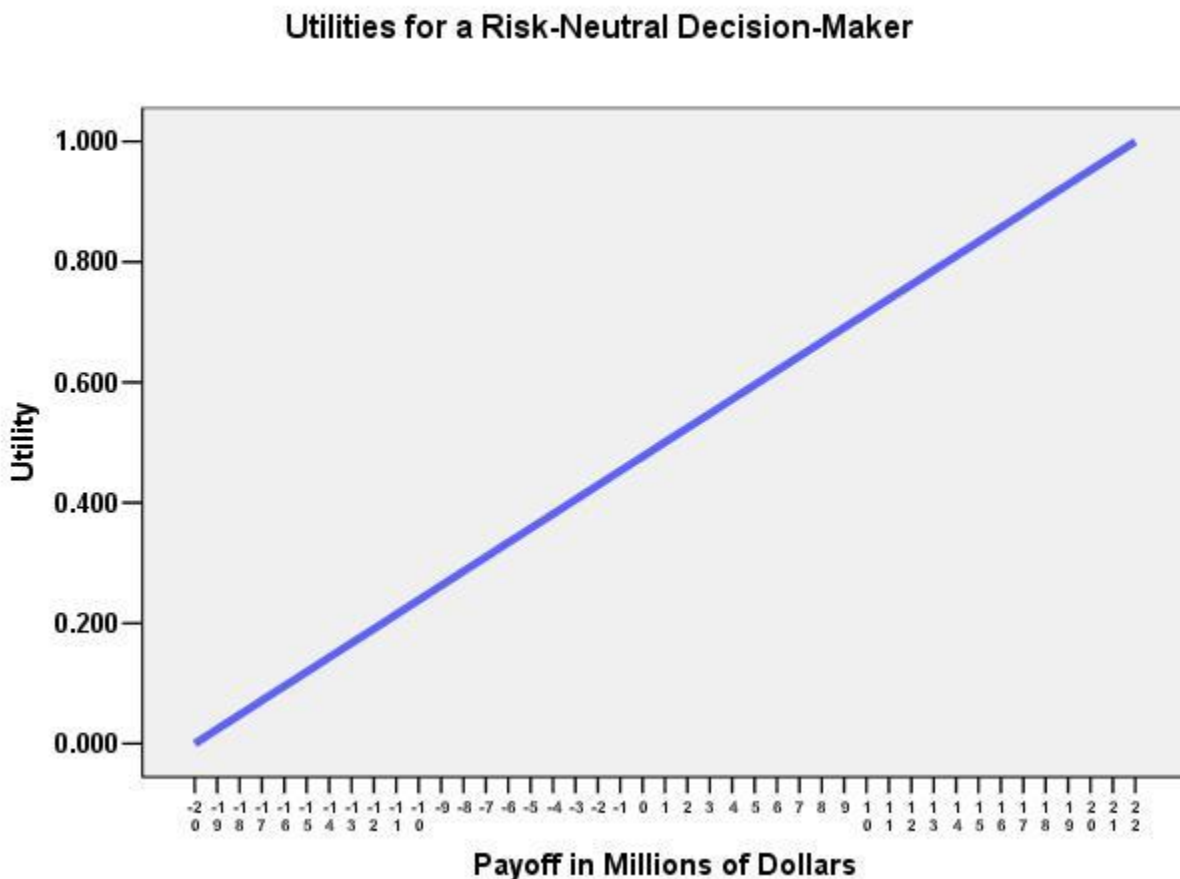
Below is the decision tree for the modified problem, where we reduce the probability that the stadium will be built in the East suburb from 0.50 to 0.45. This causes a change in our decision, from buying both parcels to buying only the West parcel, which the tree diagram clearly shows:



## Utility Theory

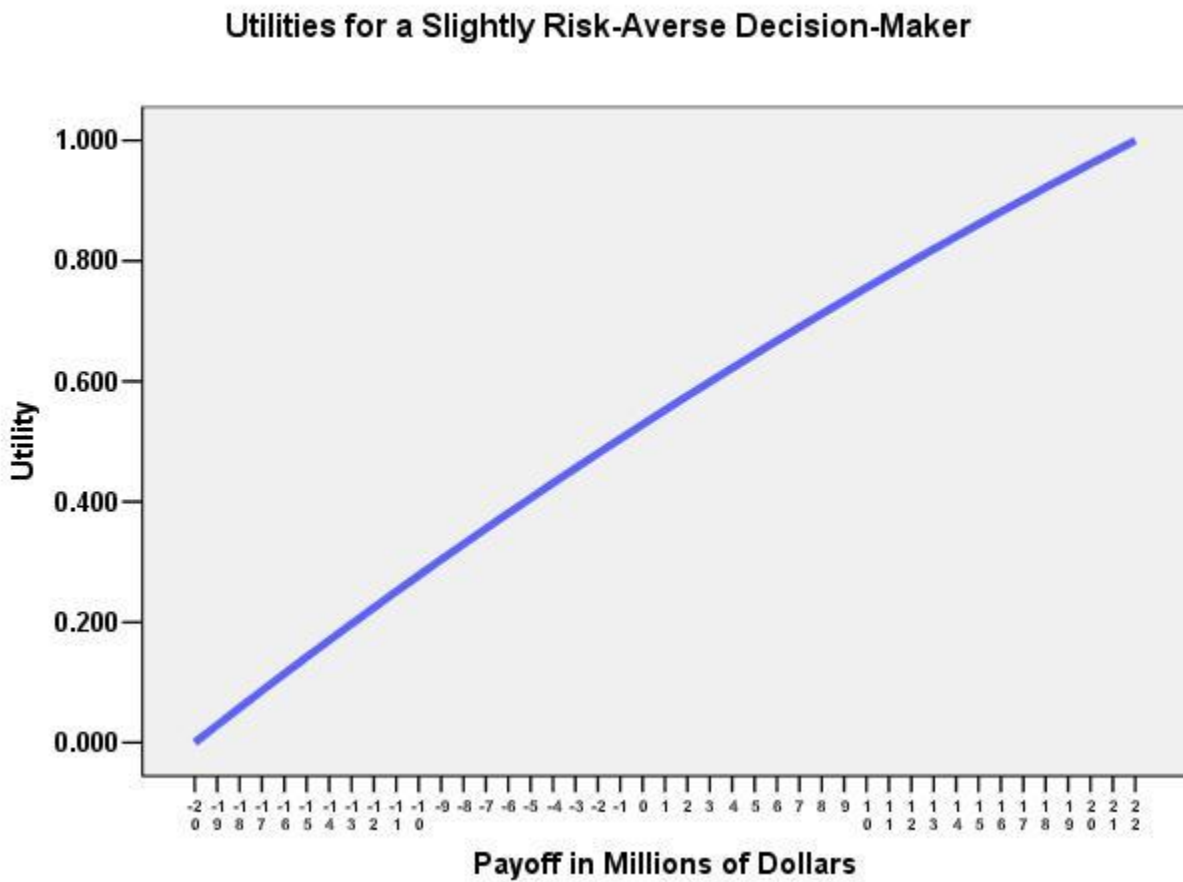
While the EMV approach to decision analysis is often useful, it does not take into account the possibility that different decision-makers may have different **risk tolerances** or **risk profiles**. Depending on the situation and/or the decision-maker, it is useful to be able to adjust the analyses to suit any of a variety of possible risk profiles. Utility theory allows us to do that.

Without getting too technical or mathematical, we'll just point out that most business decision-makers tend to be either risk-neutral or somewhat risk averse; so we'll focus on those types of decision-maker. Also, being more risk-averse means that the decision-maker is willing to settle for a smaller potential payoff in exchange for reduced risk ("lower risk, lower reward"). Here is a utility function graph for a risk-neutral person:



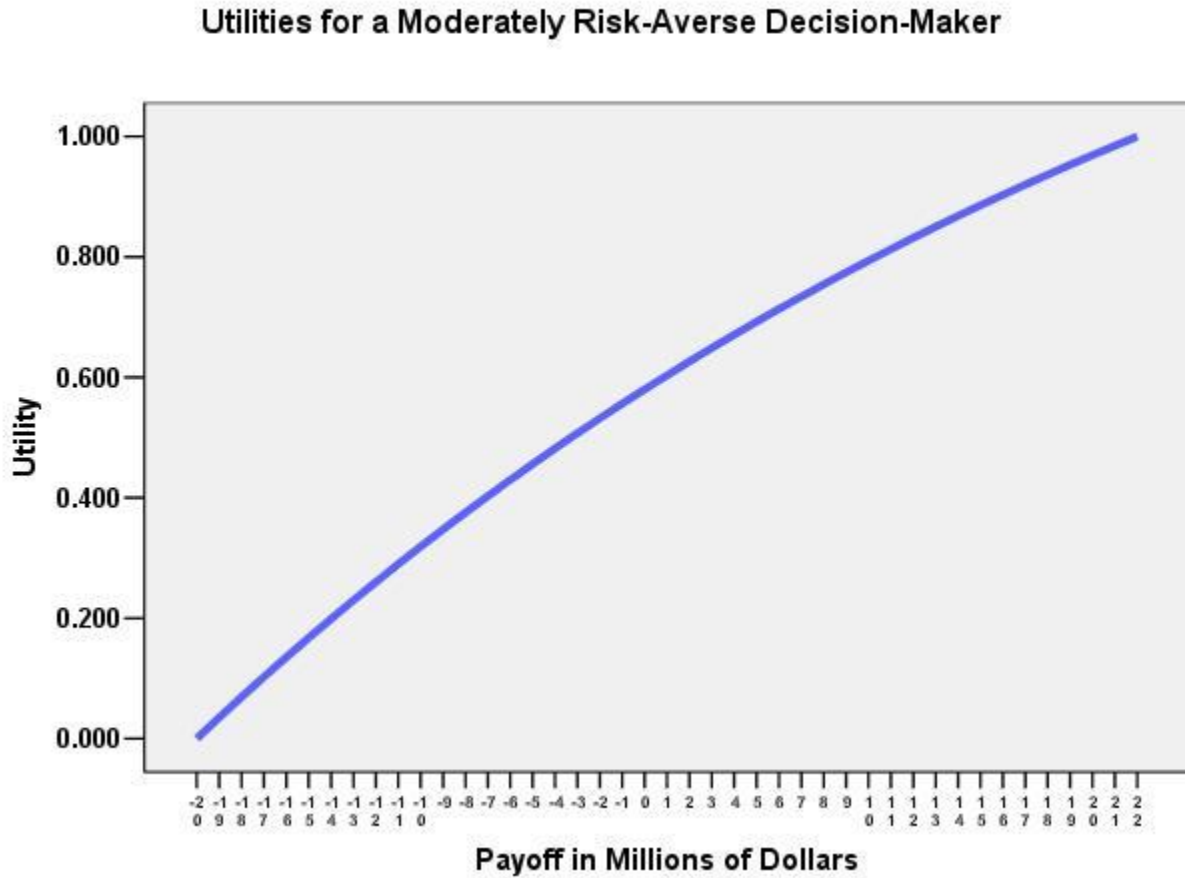
The risk-neutral person follows the EMV decision rule we discussed earlier, resulting in a linear utility function across the spectrum of possible payoffs ranging from -\$20 million to +\$22 million. The risk-neutral person is said to have constant marginal utility for increased payoffs. In other words, each additional dollar in potential payoff leads to the same amount of increase in utility.

Here is a utility function graph for a slightly risk-averse person:



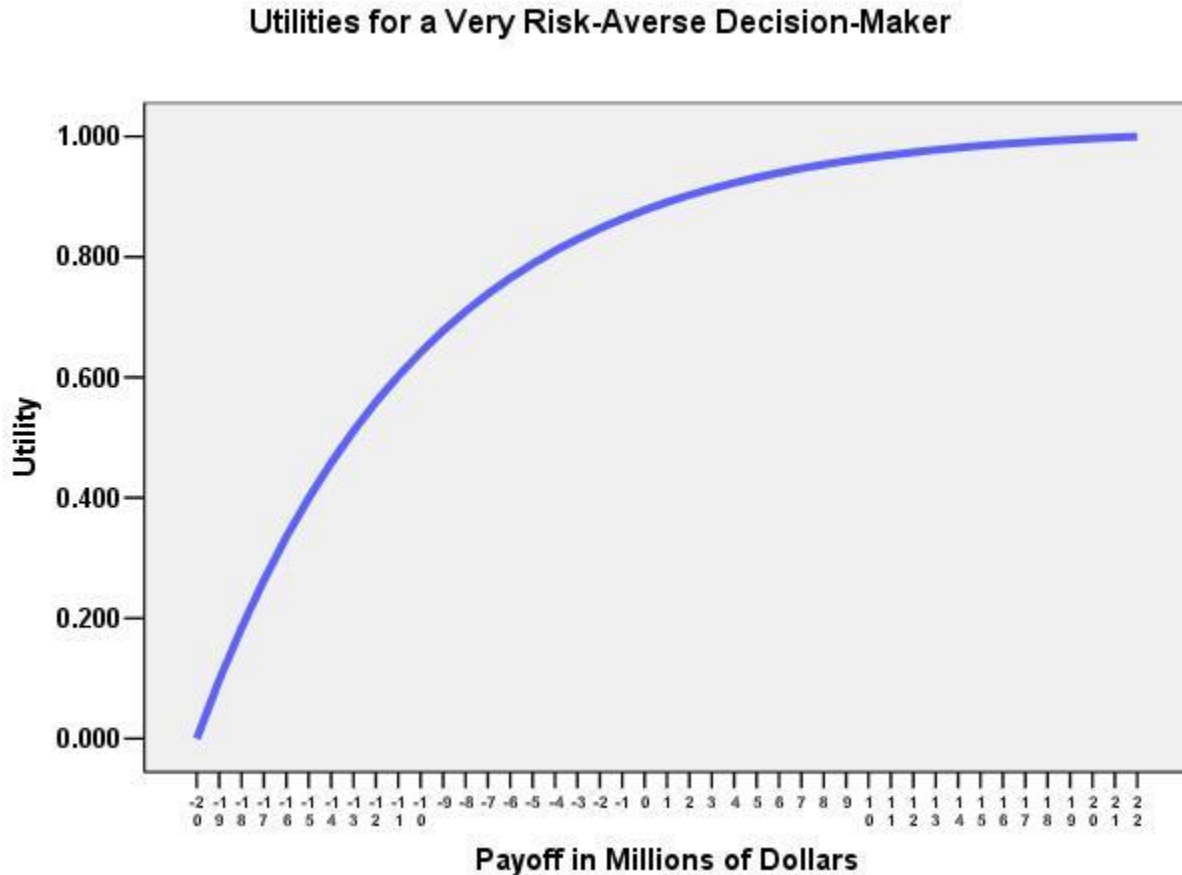
Here we can see a slightly concave utility function (a slight exponential curve), indicating a tendency for the person to have a slightly reduced marginal utility for increased profit beyond a certain point.

Next we see a utility function graph for a moderately risk-averse person:



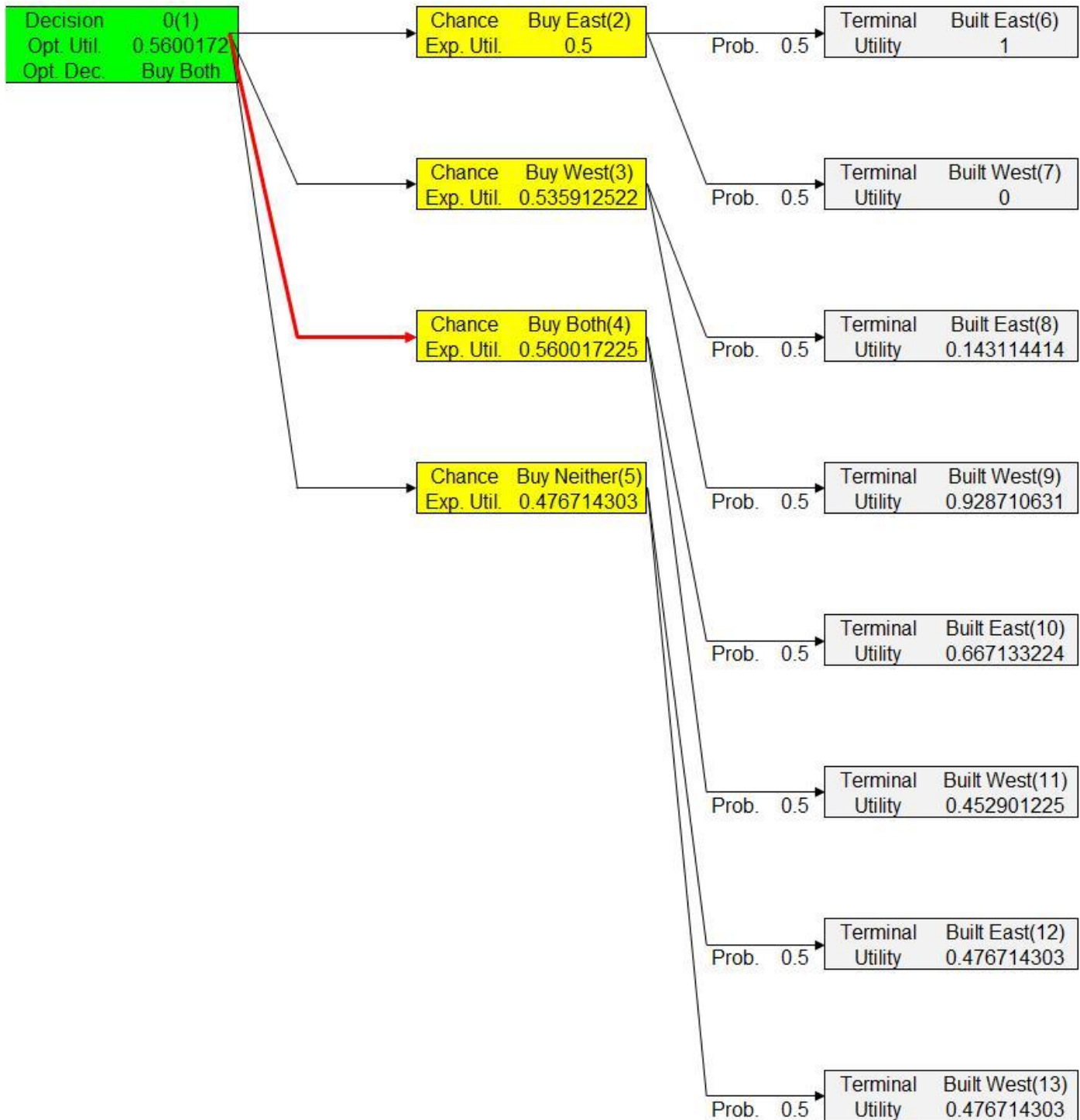
Here the person is expressing an even greater diminution in marginal utility with increasing payoffs.

And finally, here is a graph for a very risk-averse individual:



Here the decision-maker shows a marked diminution in marginal utility after potential payoffs reach a certain point. (Note: although we did not show it here, the utility function graph for a risk-taker would be convex instead of concave, indicating a nonlinearly increasing marginal utility associated with increasing payoffs.)

Now let's examine the utility-theory versions of our original decision tree diagrams. First, let's look at how the first decision tree above (that assumed a 50/50 chance of the stadium's being built at the East location) can be rolled back and portrayed according to utility theory, based on the risk profile of a risk-neutral decision-maker (i.e., someone who is neither risk-averse nor a risk-taker):

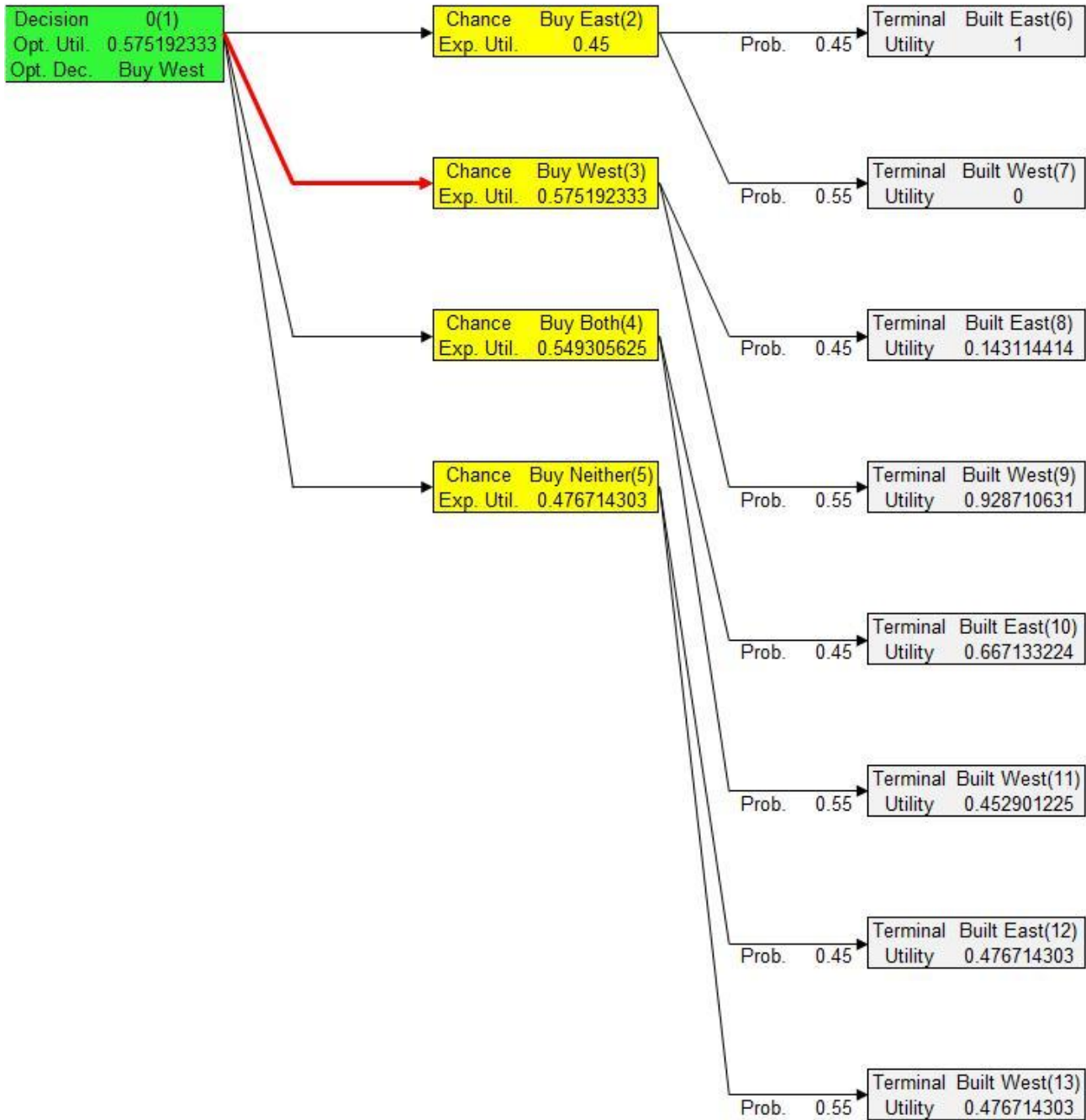


The decision tree looks somewhat similar to the first one, except that now the various boxes contain utilities instead of EMV values. (Note: for the risk-neutral decision-maker, the EMV of 3.5 associated with a 0.50 probability of the stadium's being built in the East suburb is equivalent to the utility value of 0.56 shown in the above tree. The reader can confirm this by examining the linear utility-function graph for the risk-neutral decision-maker we discussed earlier, as well as the color-coded table that appears above at the start of the **Sensitivity**



**Analysis** section of this page.)

And finally, here is the utility-theory version of the decision tree in which we assumed only a 0.45 probability that the stadium would be built in the East suburb. Here again, the model is based on a risk-neutral decision-maker:



In this case, the 0.575 utility in the second yellow box is equivalent to the 4.2 EMV value shown in the earlier decision matrix that displayed a 0.45 probability for the East location and also

showed an EVPI (expected value of perfect information) of \$16.2 million.

SmartDrill can assist our clients with very difficult decisions using any of the above techniques, although we have generally found decision trees based on utility functions to be the most useful. And although we have presented relatively simple single-stage decision trees, we can build very complicated multi-stage trees as well. The latter can help to sort out very complex decision processes, and can typically help clients save a great amount of money.

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